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Industry CPD

Accuracy and precision in finite-element analysis

This CPD module, sponsored by Oasys, explains what we mean by accuracy and precision, looks at possible causes of inaccuracy when conducting a finite-element analysis, and discusses the role of validation and verification in checking the accuracy of modelling output.

Continuing professional development (CPD) ensures you remain competent in your profession. Chartered, Associate and Technician members of the Institution must complete a specified amount each year. All CPD undertaken must be reported to the Institution annually. Reading and reflecting on this article by correctly answering the questions at the end is advocated to be:

1 hour of verifiable CPD

Why is accuracy important?

'We demand rigidly defined areas of doubt and uncertainty'

Douglas Adams[1]

As structural engineers, we must guarantee that our structures are safe while also being economical and sustainable. Structural engineers must balance these conflicting responsibilities, so we need to understand our proposed designs with as much accuracy and precision as possible.

So, how can we achieve accuracy and precision? What accuracy do we require and what do we mean by these terms?

Difference between accuracy and precision

Accuracy and precision are words that often go together, but in an engineering context they have different meanings. Accuracy is how close the answer is to the correct one[2], while precision means either the closeness of agreement among a set of results, or the number of significant figures used in the calculations[3]. For example, setting π to 3.215435881 is quite precise, but wrong; 3.14 is less precise but more accurate. Modern computers typically use 64-bit binary numbers, which gives us a precision of about 16 decimal digits[4]. However, that does not guarantee that the answers are accurate.

What is accuracy?

A model or calculation is not accurate or inaccurate, but rather is or is not sufficiently accurate. The answer to the question, 'is your watch accurate?', depends on what you want to do with it. A stopped watch is accurate twice a day, but you just don't know when: it is not useful. On the other hand, a watch that gains a minute a day is sufficiently accurate if you want to meet up with a friend.

For any real problem, a non-linear analysis will be more accurate than a linear one, but by how much, and is this additional accuracy useful? When deflections are small and materials remain elastic, then the differences between linear and non-linear results are likewise small, but when deflections are large, then they can give significantly different answers. Then there is the transition zone between the two extremes, where the expertise of the engineer is essential. The important question is not whether the model is accurate, but rather whether it gives accurate insight into how the structure works.

How much accuracy and precision do we need?

To illustrate this, consider the question: where are you? If you were to tell me where you grew up, then the city, county, or even country may be sufficiently precise: to the nearest 10, 100, or 1000km. But if we want to meet in person, then a precision of 10m is more appropriate ('I'll meet you in…, or on the corner of…'), or even 1m (on this particular table).

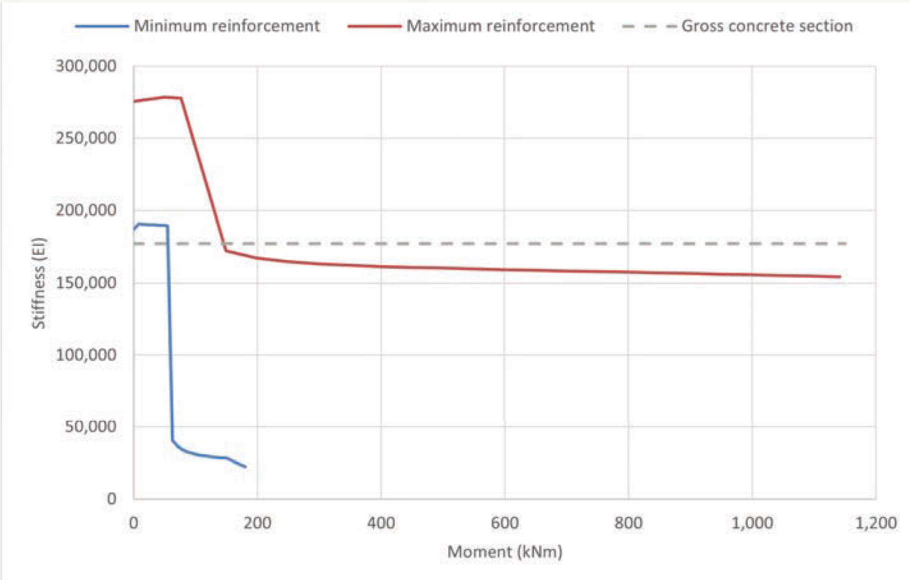
Now, if you're even more precise and give your position to the nearest 1mm, we have a different problem. The value is accurate and precise, but how valid is it? The coordinate is not now 'your position' but 'a position that is inside you'. There are many such precise coordinates that are correct, and you have specified just one of them.

It is similar with finite-element analysis (FEA). In the model, we find *one* possible stressed state, but we cannot know if it is *the* stressed state, especially when the structure is statically indeterminate (which means nearly all our structures). If the structure can behave plastically and we know that the structure can resist all likely loads, then the structure will work even though we don't exactly know how it is stressed.

Your average FEA program works to a precision of 16 significant figures, which means that if it is working in metres, the last decimal place is smaller than an atom. While the programs are helped by this level of precision in their calculations, you can safely discard most of the significant figures, typically retaining only the first three, when viewing the results.

What are the limits on accuracy?

Analysis for the design of a new-build structure is packed with unknowns that reduce the accuracy of any model. The effect of this can be seen with Hambly's (or the stool) paradox[5]. If you sit centrally on a three-legged stool, you can be sure that each stool leg will take a third



↗FIGURE 1: Example reinforced concrete beam moment stiffness chart[6]

of your weight. But if you have a four-legged stool, each leg does not carry a quarter of your weight; instead, slight imperfections in the stool and the floor mean that the stool will rock, and two legs will carry the majority of the load, but you cannot tell which in advance.

The design load in any leg of the four-legged stool can be close to half: adding a leg changes the stool from statically determinate to indeterminate (the clue is in the name) and increases the maximum member force. (Consider whether your design models take this effect into account.)

How is this expressed in real structures?

First: tolerances. While we may try to build our models perfectly, we don't know how close this will be to what is built on site. For example, the columns will not be perfectly vertical. The design codes recognise this imperfection and require us to model the effects, either by adjusting the model to lean in each of the possible directions (which is difficult without automation) or by applying some notional (i.e. approximate) horizontal loads to represent the lack of plumb.

Tolerances also affect the stiffness of beams. A typical steel beam has allowable tolerances of about 2% on its individual dimensions and about 4% of the weight (i.e. cross-sectional area and axial stiffness). Taking all these into account means that the bending stiffness of the beam can vary by over ±10%. This has two main effects. First, this variation on the stiffness means a ±10% variation on the deflection: you cannot predict the deflection better than two significant places. For example, a 15mm beam deflection will be somewhere between 13.5 and 16.5mm, so saying that it will be 15.0936mm is nonsense.

The stiffness of a concrete beam is far more variable, due to the concrete mix, the time since pouring, the actual dimensions and bar locations, the quantity of rebar and the load on it (stiffness reduces as moment increases). Even just the cracking can more than halve the stiffness **(Figure 1)**. We might iterate the analysis, analysing, choose the rebar, updating the stiffness, reanalysing, etc., but we still will not be certain of the cover, dimensions and concrete properties.

What things reduce accuracy?

Within the maths, there is the problem of delivering a single answer for an indeterminate structure. If it's a linear analysis, then there is the assumption, correct or otherwise, that the equilibrium is found at the undeformed position. For non-linear analysis, which is searching for the equilibrium at the deformed position, there is the convergence criteria to be aware of: is it close enough while still analysing in a reasonable time limit?

Accuracy can be affected by the things that we cannot know in advance:

- →| the actual stiffness of each section
- →| the actual location of each member
- →| the actual material properties
- →| the actual loads that will be experienced by the structure and how quickly they are applied
- →|the actual stressed state of the structure

and what is included in the model or not:

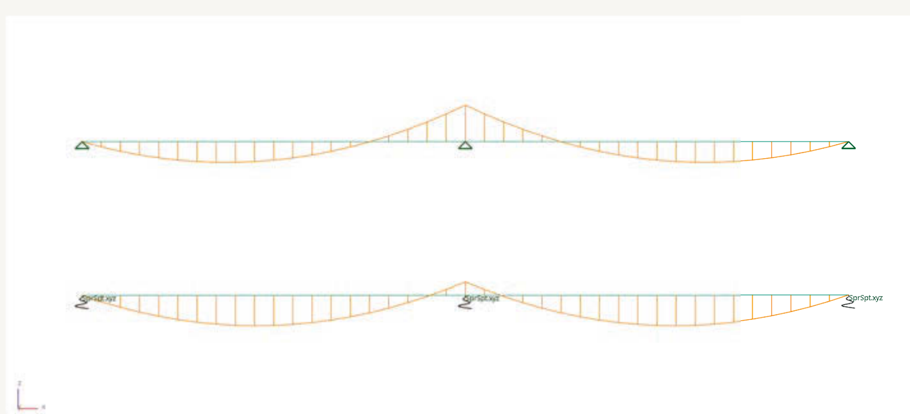
- →| support stiffnesses
- →| connection stiffnesses
- →| material yielding or non-linear stress–strain relationships.

As variations in stiffnesses redistribute moments in statically indeterminate structures, there is a limit to how accurately we can predict them. Consider the two-span concrete beam shown in **Figure 2**, which is often analysed with infinitely stiff pinned supports, followed by an arbitrary redistribution of the moments. The literature tends to talk about collapse behaviours and plastic hinges, but a major source of redistribution is the finite stiffnesses of the supports.

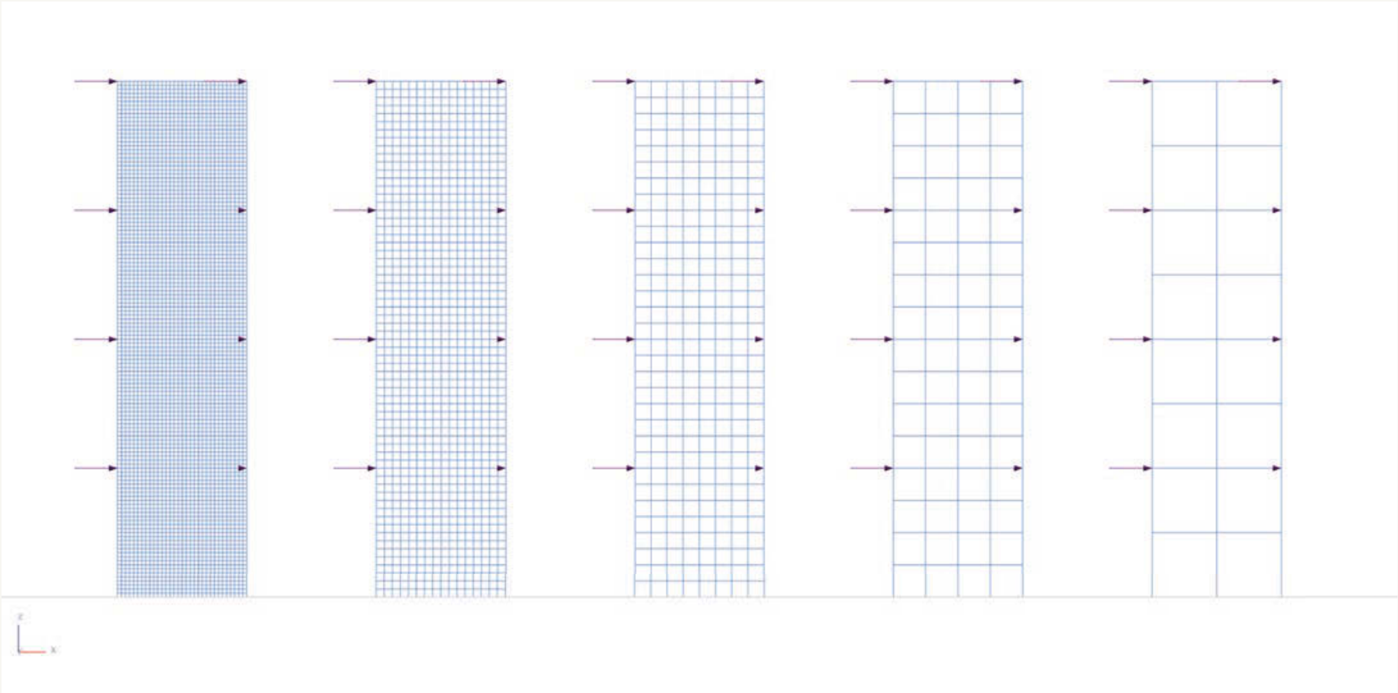
Analyse the beam with realistic supports and the moments automatically redistribute from the mid-point to the spans **(Fig. 2)**. What the support stiffness is depends on several factors, including foundation settlement and column axial shortening.

Finally, there is what is not output by the analysis because it was not asked for or not available. What was missing was crucial for the historic collapses outlined below[8,9].

- →| **Sleipner A oil rig base, North Sea:** 3D elements gave shear stresses not shear forces. Inappropriate modelling combined with lack of checking underestimated the shear force by 47%.
- →| **Hartford Civic Center, Connecticut:** the model did not reflect what was actually built and the analysis did not check buckling capacity. This was combined with connections that induced torsional moments and sections with minimal torsional stiffness.
- →| **CTV Building, Christchurch, New Zealand:** seismic analysis captured the



↗FIGURE 2: Two-span concrete beam with and without spring supports[7]



↗FIGURE 3: Core walls with varying mesh size[7]

storey drifts of the centre of the building, but because of the asymmetric stability system, the corner drifts were significantly higher than those used in the design.

Modelling causes of inaccuracy

Remember that you are analysing not the real structure but a model, which is, by necessity, a simplification. How you define and analyse your model will have a major effect on its accuracy. This may include your choice of 1D, 2D or 3D elements, how the connections and joints are represented, and how small you break down the elements.

Consider how the mesh refinement affects the analysis results. If you take the example of a core shear wall with loading at each storey **(Figure 3)**, and vary both the mesh size (starting at two elements per storey and halving each time) and the element type (Linear / Quad4 and Quadratic / Quad8), then we can see that smaller and higher-order elements are considerably more accurate **(Figure 4).**

The effect of mesh size on buckling analysis can be even greater. A rule of thumb in FEA is to half the element size, reanalyse, and then stop when you can get no further improvement in the results. Do though watch out for stress concentrations (such as at support points and corners in openings), where refining the element size will lead to continually increasing stresses in these areas. Also, because higher-order elements are more accurate, you get to the accuracy needed with fewer elements.

(Note that higher-order element formulations can have reduced degrees of freedom, so ensure that they are still valid for your problem.)

Mathematical causes of inaccuracy

FEA requires a very large number of calculations using floating point numbers, but the results do not have the precision implied by the number of bits used to store those numbers. All the forces and moments are derived from the element strains, which are in turn derived from the difference in movement of the element ends. If the overall translation of the element is large and/or the element coordinates are large, then the strains can easily lose several digits of precision. This in turn limits the accuracy of the results.

Other reasons why model accuracy can reduce include:

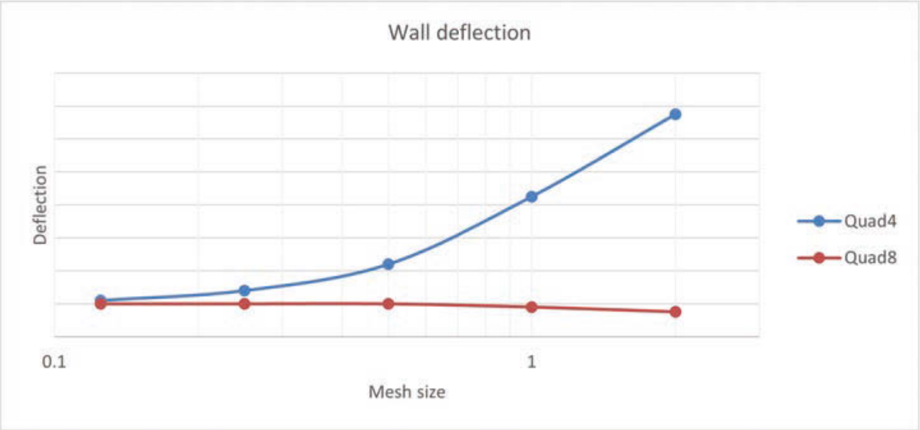
- →| **a finer mesh density:** this can improve the accuracy of the analysis but may also increase the rounding error.
- →| **the range of stiffnesses in the model:** the closer the maximum and minimum element stiffnesses are together, the better. Sometimes it can be better to break up long slender elements into smaller pieces, or to combine short stocky elements. If they are particularly stiff, consider replacing them with links or rigid constraints.
- →| **distance of the model from the origin:** the more digits you have to the left of the decimal point, the less precision you have to the right. It might pay to move your model if it has been created from BIM geometry.

How do we know if the results are sufficiently accurate?

'…there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns – the ones we don't know we don't know.'

Donald Rumsfeld[10]

All results must be checked to ensure accuracy, by assessing the validity of both the model



↗FIGURE 4: Effect of mesh size and element type on deflection and the software, then *verifying* that the results are correct.

Validation means: are you asking the right question? What is it that you are trying to figure out or understand about a particular engineering problem? Do you need to assess the static linear behaviour of the structure, or do you need your model to take non-linear effects or buckling into account? Do you need to consider the dynamic behaviour?

Is the software capable of doing such calculations? Have you built the model to give you the correct behaviour?

Essentially, validation means whether you have built your analytical model in accordance with the initial design intent and whether the subsequent detailed design is in accordance with the analytical model.

Verification means: are you getting the right answers? Are there any errors in either the model or software? Commercial software has layers of testing to ensure that it does the calculations correctly, but you still need to have informed confidence in the program. It is your responsibility to deliver the correct results, so evaluate the software yourself.

Verification of the model brings us to a paradox of FEA: you need to know the answers before running the model. While engineering experience is invaluable, quantitative analysis (determining the deflected shape and bending moment diagram in advance[11]), a simpler model, or using standard beam/frame formulas (such as $wl^2/8$) can all be useful.

If the answers are close to each other (precision), then that should give you some confidence in the correctness (accuracy), but remember that you may have made the wrong assumptions in both the model and the check calculation. This is where a second pair of eyes, owned by a more experienced engineer, is essential.

Conclusion

If there are limits to the accuracy of FEA, how do we know if it is accurate and why should we use it rather than manual calculations? The answer is that, done correctly, FEA will usually be more accurate, as well as considerably quicker. But, as the adage has it, 'garbage in, garbage out': all calculations can be spectacularly wrong if your inputs are also wrong.

While FEA is a standard analysis tool in structural engineering, the accuracy will depend on the fidelity of the model: inaccuracies come into the model from multiple sources. It is up to you, as the engineer, to ensure that your model is adequate and that the results are sufficiently accurate for purpose.

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Questions

To claim your CPD certificate, answer the questions online at **www.istructe.org/industry-cpd**. The module will close on 7 January 2024.

1) About how accurately can you predict the deflection of a steel beam?
a) 100%
b) ±100%
c) ±10%
d) ±0%

2) What is the maximum reasonable precision for a person's location?
a) 10m
b) 1m
c) 1cm
d) 1mm

3) How do you know if your mesh is sufficiently fine?
a) I used the software defaults.
b) I made the elements as large as possible.
c) I made the elements as small as possible.
d) I kept halving the mesh size until there was no significant change in the results.

4) What is the relationship between accuracy and precision?
a) Accuracy requires precision.
b) Precision requires accuracy.
c) Precision is more important than accuracy.
d) Precise data implies accurate results.

5) What are validation and verification?
a) Validation is when your boss thinks you are doing an excellent job and verification is that your team agree.
b) Validation means you are asking the right question in your model and verification means that it is giving sufficiently accurate answers.
c) Validation means that your software can address your problem and verification is that it gives accurate answers.
d) Both answers (b) and (c) above.

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